

12. (a) (i) Find the Fourier series of $f(x) = x^2$ in $-\pi < x < \pi$. Hence deduce the value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$. (8)

- (ii) Find the half range cosine series expansion of $(x-1)^2$ in $0 < x < 1$. (8)

Or

- (b) (i) Compute the first two harmonics of the Fourier series of $f(x)$ from the table given. (8)

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
$f(x)$	1.0	1.4	1.9	1.7	1.5	1.2	1.0

- (ii) Obtain the Fourier cosine series expansion of $f(x) = x$ in $0 < x < 4$.

Hence deduce the value of $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots$ to ∞ . (8)

13. (a) If a tightly stretched string of length l is initially at rest in equilibrium position and each point of it is given the velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = v_0 \sin^3 \frac{\pi x}{l}$, $0 < x < l$, determine the transverse displacement $y(x, t)$. (16)

Or

- (b) A square plate is bounded by the lines $x = 0$, $x = a$, $y = 0$ and $y = b$. Its surfaces are insulated and the temperature along $y = b$ is kept at 100°C . while the temperature along other three edges are at 0°C . Find the steady - state temperature at any point in the plate. (16)

14. (a) Find the Fourier transform of $f(x) = \begin{cases} 1 - |x|, & \text{if } |x| < 1 \\ 0, & \text{otherwise} \end{cases}$. Hence, deduce the

values (i) $\int_0^{\infty} \frac{\sin^2 t}{t^2} dt$ (ii) $\int_0^{\infty} \frac{\sin^4 t}{t^4} dt$. (16)

Or

- (b) (i) Find the Fourier transform of $e^{-a^2 x^2}$, $a > 0$. Hence show that $e^{-\frac{x^2}{2}}$ is self reciprocal under the Fourier transform. (8)

(ii) Evaluate $\int_0^{\infty} \frac{dx}{(x^2 + a^2)(x^2 + b^2)}$, using Fourier transforms. (8)

15. (a) (i) Find $Z(\cos n\theta)$ and $Z(\sin n\theta)$. (8)

(ii) Using Z -transforms, solve $u_{n+2} - 3u_{n+1} + 2u_n = 0$ given that $u_0 = 0, u_1 = 1$. (8)

Or

- (b) (i) Find the Z -transform of $\frac{1}{n(n+1)}$, for $n \geq 1$. (8)

(ii) Find the inverse Z -transform of $\frac{z^2 + z}{(z-1)(z^2+1)}$, using partial fraction. (8)

